## Shear strength of ceramics

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Nanoindentation experiments [1, 2], Bragg-Nye bubble-raft simulation experiments [3], and atomistic/ finite-element modeling [4] have revealed that incipient plasticity in ductile metals occurs at shear stress values approaching the theoretical shear strength. This incipient plasticity, characterized by a sudden burst of displacement in the nanoindentation load-depth (P-h) response at sub-milliNewton loads (0.01 to 0.03 mN for single-crystal Al) [2], represents homogeneous nucleation of defects such as dislocations. In order to investigate if incipient plasticity can be detected quantitatively in brittle ceramics using displacement bursts in nanoindentation P-h responses, we have performed low-load nanoindentation experiments on two singlecrystals: Y<sub>2</sub>O<sub>3</sub> and SrTiO<sub>3</sub>, cubic ceramics whose shear moduli differ by about a factor of two. We have found that displacement bursts in the nanoindentation P-h responses occur at average indentation loads of 0.15 and 0.65 mN in Y<sub>2</sub>O<sub>3</sub> and SrTiO<sub>3</sub>, respectively, and that these loads are consistent with the calculated theoretical shear strengths of the two ceramics.

Single-crystal SrTiO<sub>3</sub> (100) was obtained commercially (Material-Technologie & Kristalle Gmbh, Germany), whereas a polished large-grain size (100  $\mu$ m) Y<sub>2</sub>O<sub>3</sub> polycrystalline specimen was obtained from Dr. W. H. Rhodes. While the (100) surface of the SrTiO<sub>3</sub> single-crystal was indented, the orientation of the indented grain in the Y<sub>2</sub>O<sub>3</sub> was unknown. Approximately 50 nanoindentation experiments were performed on each material at different surface locations, using Nanoindenter XP (MTS/Nanoinstrument, Oak Ridge, TN), in Dynamic Contact Module<sup>TM</sup> (DCM) mode, equipped with a diamond Berkovich pyramid. The peak load ranged from 0.1 to 1.0 mN, while the total loading time required to reach the peak load was maintained constant at 30 s.

Fig. 1A and B show representative *P*-*h* curves for the nanoindentation of  $Y_2O_3$  and SrTiO<sub>3</sub>, respectively. Note the displacement bursts at indentation loads (*P*<sup>\*</sup>) of 0.145 and 0.625 mN in  $Y_2O_3$  and SrTiO<sub>3</sub>, respectively. The indentation loads at *P*-*h* discontinuities for  $Y_2O_3$  ranged from 0.1 to 0.2 mN, with an average of *P*<sup>\*</sup> = 0.15 mN. In the case of SrTiO<sub>3</sub>, *P*<sup>\*</sup> ranged between 0.5 and 0.8 mN, with an average of *P*<sup>\*</sup> = 0.65 mN. A hard "dummy" specimen (SiC) was nanoindented several times (at different locations) under the same conditions, where no displacement bursts were observed. This confirmed that the observed dis-

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placements bursts in  $Y_2O_3$  and  $SrTiO_3$  are not an artifact of the instrument, and that those bursts arise from material response alone.

The maximum shear stress under the Berkovich indenter at indentation load  $P^*$ , where the *P*-*h* discontinuity occurs, can be determined using the following relation [2, 5]:

$$\tau_{\rm Max} = 0.31 \left( \frac{6P * E^{*^2}}{\pi^3 R^2} \right)^{0.33},\tag{1}$$

where *R* is the radius of the tip of the indenter, and  $E^*$  is defined in terms of the Young's moduli and the Poisson's ratios of the diamond indenter ( $E_i$ ,  $v_i$ ) and the specimen ( $E_s$ ,  $v_s$ ) as follows [2, 5]:

$$E* = \left(\frac{1 - \nu_{\rm s}^2}{E_{\rm s}} + \frac{1 - \nu_{\rm i}^2}{E_{\rm i}}\right)^{-1}.$$
 (2)

Although the Berkovich pyramid is a "sharp" indenter, it has a finite radius, which is assumed to be  $R \sim 50$ nm for a new indenter [2]. Using the values for elastic properties of diamond, Y<sub>2</sub>O<sub>3</sub>, and SrTiO<sub>3</sub> (Table I), the maximum shear stress values  $\tau_{Max}$  at *P*-*h* discontinuities for Y<sub>2</sub>O<sub>3</sub> and SrTiO<sub>3</sub> are calculated to be 16.1 and 34.1 GPa, respectively (Table I). A rough estimate for the theoretical shear strength of crystals is given by [6]:

$$\tau_{\rm Th} = G/2\pi,\tag{3}$$

where G is the shear modulus of the specimen, which is given by [6]:

$$G = \frac{E_{\rm s}}{2(1+\nu_{\rm s})}.\tag{4}$$

Using Equations 3 and 4, the theoretical shear strengths for Y<sub>2</sub>O<sub>3</sub> and SrTiO<sub>3</sub> are calculated, and they are also given in Table I, along with  $\tau_{Max}/\tau_{Th}$  ratios. Although the estimate of the theoretical shear strength is only an approximation (Equations 3) [6], these ratios are not significantly different compared with unity (1.5 and 1.7), indicating that  $\tau_{Max}$  represents the theoretical shear strength of the ceramic in question. This notion is further reinforced by the fact that the  $\tau_{Max}/\tau_{Th}$  ratios for the two ceramics are not very different. The local breakdown of the material could be manifest as fracture or

TABLE I Relevant mechanical properties of ceramics, and calculated values of  $\tau_{Max}$  (Equation 1) and  $\tau_{Th}$  (Equation 3)

Ceramic	<i>P</i> * (GPa)	E (GPa)	ν	G (GPa)	$\tau_{\rm Max}~({\rm GPa})$	$\tau_{\mathrm{Th}}~(\mathrm{GPa})$	$ au_{ m Max}/ au_{ m Th}$
Y <sub>2</sub> O <sub>3</sub>	0.15	172 [12]	0.31 [12]	66 [12]	16.1	10.5	1.5
SrTiO <sub>3</sub>	0.65	300 [13]	0.20 [13]	125 [13]	34.1	19.9	1.7
Diamond	_	1000 [2]	0.07 [2]	_	_	-	_
MgO	1.0 [7]	290 [11]	0.19 [11]	130 [11]	38.5	19.4	2.0
Al <sub>2</sub> O <sub>3</sub>	15 [9]	400 [10]	0.22 [10]	164	110.8	26.1	4.2



*Figure 1* Representative nanoindentation P-h responses (loading) for single-crystals of: (A)  $Y_2O_3$  and (B) SrTiO<sub>3</sub>.

dislocation creation [4], the exact nature of which is not clear at this time.

In this context, Gaillard *et al.* [7] have recently observed similar displacement bursts during the nanoindentation of another cubic ceramic, MgO. For singlecrystal MgO (001) specimens, the indentation load at which this burst occurs ( $P^*$ ) is 1.0 mN. However, the displacement burst in terms of indentation depth (h) in MgO (25 nm) [7] was found to be significantly greater than that observed for Y<sub>2</sub>O<sub>3</sub> (3 nm) and SrTiO<sub>3</sub> (7 nm). Using Equations 1 to 4 and elastic-property values (Table I) in the case of MgO, the  $\tau_{Max}/\tau_{Th}$  ratio was found to be 2.0, which, once again, is consistent with

1892

 $\tau_{Max}$  identifying with the theoretical shear strength of MgO.

Gaillard *et al.* [7] have also performed careful etching experiments on nanoindented MgO, which is a way to reveal dislocations that intersect the free surface [8], and they have characterized the resulting etch-pits using the atomic force microscope (AFM). For nanoindentation loads greater than  $P^*$ , they have found well-defined patterns of etch-pits around the nanoindentation site [7]. For  $P < P^*$ , the etching results were inconclusive [7]. These results suggest that the displacement bursts in nanoindentation of cubic ceramics are associated with the nucleation of dislocations.

Page *et al.* [9] were the first to report nanoindentation displacement bursts in ceramics. However, they found the burst to occur at much higher loads ( $P^*$  for single-crystal Al<sub>2</sub>O<sub>3</sub> ~ 15 mN). Using Equations 1 to 4 and elastic-property values (Table I) in the case of rhomohedral Al<sub>2</sub>O<sub>3</sub>, the  $\tau_{Max}/\tau_{Th}$  ratio is found to be 4.2, which is higher than that for cubic ceramics. This could be possibly due to the reduced crystal symmetry in Al<sub>2</sub>O<sub>3</sub>. Page *et al.* [9] have also attributed these bursts to the formation of incipient cracks in Al<sub>2</sub>O<sub>3</sub>.

In the case of metals, multiple displacement bursts have been observed, which occur at successively higher nanoindentaion loads P, and over larger h ranges [2]. It has been shown that this is due to the motion of the already formed dislocations, which requires orders of magnitude lower shear stresses (identifies with the Peierls stress) relative to theoretical shear strengths. In the nanoindentation of brittle ceramics, such multiple displacement bursts are not common, possibly due to the very high values of the Peierls stress relative to ductile metals [10, 11].

In summary, the displacement bursts we have observed during the nanoindentation of ceramics arise as a result of material response, and are not an instrument artifact. In the case of cubic ceramics, the maximum shear stress ( $\tau_{Max}$ ) associated with the nanoindentation load at which these bursts occur identifies with the respective theoretical shear strength ( $\tau_{Th}$ ) of the ceramic in question. It appears that homogenous nucleation of dislocations is responsible for the displacement burst, however, further work is needed to elucidate this issue.

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